The Paretian Optimum

Mathematical Derivation of the Conditions:

We may also derive mathematically the marginal condition for Pareto efficiency in production.

Let us suppose that the production functions for the goods Q_1 and Q_2 are:

$q_1 = q_1 (x_{11}, x_{12})$

and $q_2 = q_2 (x_{21}, x_{22})$

where q_1 and q_2 are the quantities produced of goods Q_1 and Q_2, x_{11} and x_{12} are the quantities of inputs X_1 and X_2 used in the production of Q_1, and x_{21} and x_{22} are the quantities of these inputs used in the production of good Q_2.

Since the total available quantities of the two inputs are x^0_1 and x^0_2, we may write:

As per the requirements of Pareto optimality, the efficiency conditions may be derived if we maximise q_1 as given by subject to:

where q^0_2 is any given quantity of good Q_2.

The relevant Lagrange function for this constrained maximisation problem is:

The first order or the necessary conditions for maximum q_1 subject to $q_2 = q^0_2$ are:

Pareto efficiency condition gives us that the available quantities of the two inputs, X_1 and X_2, should be allocated over the production of the two goods, Q_1 and Q_2, in such a way that the MRTS between the inputs may be the same in the production of the two goods.

We may now see with the help of a simple example why condition (21.7) is necessary for Pareto efficiency in production. Let us suppose that in the production of Q_1, $MRTS_{X1,X2} = 2$ and, in the production of Q_2, $MRTS_{X1,X2} = 1$

i.e., the MRTS is not the same in the production of the two goods.

It follows from above that we can substitute 1 unit of X_1 for 2 units of X_2 in the production of Q_1, and keep the output of Q_1 constant. Similarly, we can substitute 1 unit of X_1 for 1 unit of X_2 in the production of Q_2, and keep the output of Q_2 constant. So, all we have to do is to take 1 unit of X_1 out of the production of Q_2 and use it in the production of Q_1.