S. S. College. Jehanabad (Magadh University)

Department : Physics

Subject : Quantum Mechanics

Class : B.Sc(H) Physics Part III

Topic: Potential Step

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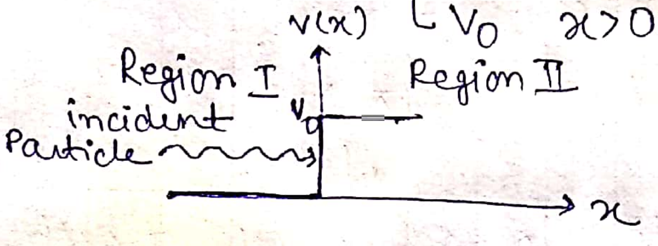
The Potential step

It is an infinite width potential barrier given by

$$V(x) = \begin{cases} 0 & x<0 \\ V_0 & x>0 \end{cases}$$



If $E<V_0$, then Particle reflected back at $x=0$ because it does not have sufficient energy to climb the barrier.

and on the other hand,

If $E>V_0$, then the particle would not be reflected it would keep moving towards the right with reduced energy.

The time independent schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

case I : $E>V_0$

In region I $(x<0)$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x) \Rightarrow \frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$$

$k^2 = \frac{2mE}{\hbar^2}$; The general solution is

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

In region II $(x>0)$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V_0\psi = E\psi \Rightarrow \frac{d^2\psi(x)}{dx^2} + k'^2\psi(x) = 0$$

$$(k')^2 = \frac{2m(E-V_0)}{\hbar^2}$$

The general solution is

$$\psi(x) = Ce^{ik'x} + De^{-ik'x}$$

The 'D' term is discarded as the wave in region II is not reflected.

Thus the complete eigenfunction is given by

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x<0 \\ Ce^{ik'x} & x>0 \end{cases}$$

Now from the continuity of ψ and $\frac{d\psi}{dx}$ at $x=0$

$$A+B = C \quad -(1)$$

$$k(A-B) = k'C \quad -(2)$$

from ① & ②

$$\frac{B}{A} = \frac{k-k'}{k+k'} \Rightarrow \frac{C}{A} = \frac{2k}{k+k'}$$

The reflection coefficient, R

$$R = \left|\frac{B}{A}\right|^2 = \left(\frac{k-k'}{k+k'}\right)^2 = \left[\frac{1-(1-V_0/E)^{1/2}}{1+(1-V_0/E)^{1/2}}\right]^2$$

The transmission coefficient, T

$$T = \frac{k'}{k}\left|\frac{C}{A}\right|^2 = \frac{4kk'}{(k+k')^2} = \frac{4(1-V_0/E)^{1/2}}{[1+(1-V_0/E)^{1/2}]^2}$$

Note that R and T depend only on the ratio V_0/E. It is also $R+T=1$ as it must be because the probabity is conserved.

Case 2: $E < V_0$

In region I, the schrodinger equation and its solutio[n] remain the same as in case 1 ($E > V_0$).

In region II,

$$\frac{d^2\psi(x)}{dx^2} - k^2\psi(x) = 0 \quad ; \quad k_1^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

since $V_0 > E$,

$$\psi(x) = Ce^{-k_1 x} + De^{k_1 x}$$

Now, the wave function should not become infinite at $x \to \infty$. Since $\exp(k_1 x)$ diverges in that limit. The complete eigen function is given by:

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < 0 \\ Ce^{-k_1 x} & x > 0 \end{cases}$$

$\Rightarrow$ we note that the wave function is not zero in the classically forbidden region II, although it decreases rapidly as x increases. Thus, there is a finite, though small probability of finding the particle in region II. This phenomenon is called barrier penetration and observed experimentally in various atomic and nuclear systems.