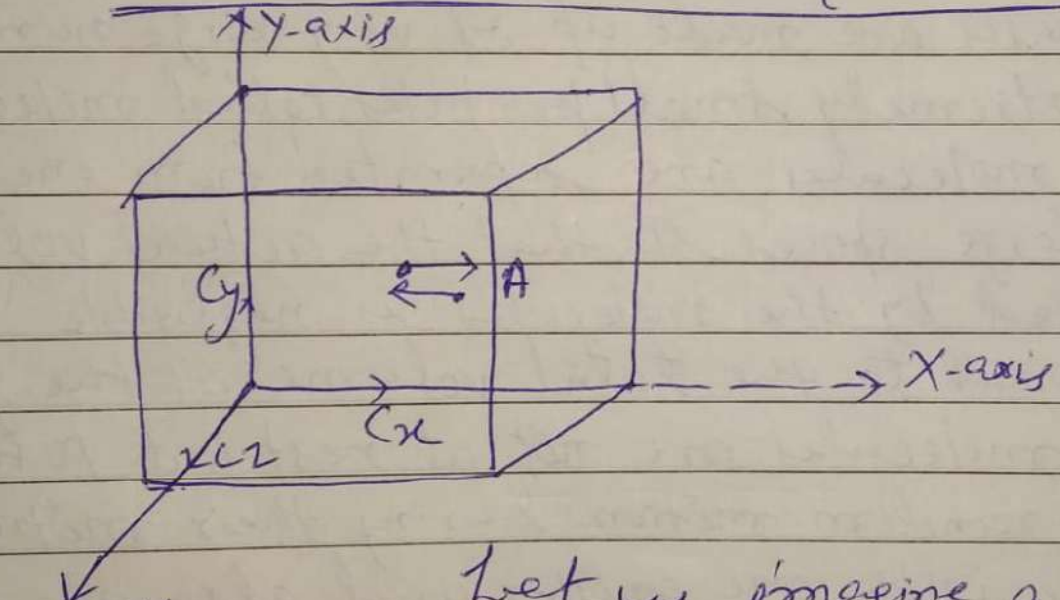
Derivation of kinetic Equation of gases



Let us imagine a given mass of a gas to be confined in a cubical vessel.

Let, l = length of the edge of the cube

n = Total no of molecules enclosed

m = mass of each molecule

C = root mean square velocity

C_x, C_y, C_z are three components related to C as –

$$C^2 = C_x^2 + C_y^2 + C_z^2$$

Let us consider only one molecule moving between two opposite faces
The velocity of the molecule before striking the face A is c_x and since it is perfectly elastic, it rebounds with the same velocity $(-c_x)$.

Momentum of the molecule before it strikes $= mc_x$

" " " after impact $= -mc_x$

Change of momentum after each impact

$= mc_x - (-mc_x) = 2mc_x$

Now the molecule strikes the same face after travelling a distance $2l$ cms with a velocity c_x cms per second.

∴ Number of impacts per second on the same face $= \frac{c_x}{2l}$

∴ Number of impacts per second on the two opposite faces along the X-axis $= 2 \times \frac{c_x}{2l} = \frac{c_x}{l}$

Hence, the total change of momentum per second due to the impact of one molecule on two opposite walls of the cube along X-axis $= 2mc_x \times \frac{c_x}{l} = \frac{2mc_x^2}{l}$

Similarly total change of momentum per second due to the impact of one molecule on the two opposite faces along Y-axis is $\frac{2mc_y^2}{l}$ and along Z axis it $\frac{2mc_z^2}{l}$

Hence the total change of momentum on on all the six faces of the cube per second per molecule —

$$= \frac{2mc_x^2}{l} + \frac{2mc_y^2}{l} + \frac{2mc_z^2}{l}$$

$$= \frac{2m}{l}(c_x^2 + c_y^2 + c_z^2) = \frac{2mc^2}{l}$$

where c is called root mean square velocity.

∴ Total change of momentum due to n molecules —

$$= \frac{2mc^2}{l} \times n = \frac{2mnc^2}{l}$$

But the change of momentum per second is force and force per unit area is pressure.

$$\therefore P = \frac{\text{Force}}{\text{Area}} = \frac{\frac{2mnc^2}{l}}{l \times \frac{6l^2}{3}}$$ (since cube has six faces)

$$= \frac{1}{3}\frac{mnc^2}{l^3} = \frac{1}{3}\frac{mnc^2}{V}$$ (∵ volume of cube $= l^3$)

$$\therefore \boxed{PV = \frac{1}{3}mnc^2}$$ This equation is called kinetic equation of gas.